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GIFFT: A Fast Solver for Modeling Sources in a Metamaterial Environment of Finite Size

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GIFFT: A Fast Solver for Modeling Sources in a Metamaterial Environment of Finite Size

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I. Introduction

Due to the recent explosion of interest in studying the electromagnetic behavior of large (truncated) periodic structures such as phased arrays, frequency-selective surfaces, and metamaterials, there has been a renewed interest in efficiently modeling such structures. Since straightforward numerical analyses of large, finite structures (i.e., explicitly meshing and computing interactions between all mesh elements of the entire structure) involve significant memory storage and computation times, much effort is currently being expended on developing techniques that minimize the high demand on computer resources.

One such technique that belongs to the class of fast solvers for large periodic structures is the GIFFT algorithm (Green's function interpolation and FFT), which is first discussed in [1]. This method is a modification of the adaptive integral method (AIM) [2], a technique based on the projection of subdomain basis functions onto a rectangular grid. Like the methods presented in [3]-[4], the GIFFT algorithm is an extension of the AIM method in that it uses basis-function projections onto a rectangular grid through Lagrange interpolating polynomials. The use of a rectangular grid results in a matrix-vector product that is convolutional in form and can thus be evaluated using FFTs. Although our method differs from [3]-[6] in various respects, the primary differences between the AIM approach [2] and the GIFFT method [1] is the latter's use of interpolation to represent the Green's function (GF) and its specialization to periodic structures by taking into account the reusability properties of matrices that arise from interactions between identical cell elements.

The present work extends the GIFFT algorithm to allow for a complete numerical analysis of a periodic structure excited by dipole source, as shown in Fig 1. Although GIFFT [1] was originally developed to handle *strictly* periodic structures, the technique has now been extended to efficiently handle a *small number of distinct element types*. Thus, in addition to reducing the computational burden associated with large periodic structures, GIFFT now permits modeling these structures with source and defect elements. Relaxing the restriction to strictly identical periodic elements is, of course, useful for practical applications where, for example, a dipole excitation may be of interest or, as is often the case for metamaterials, defective elements are introduced in the structure's fabrication process. The main extensions of the GIFFT method compared to [1] are the following:

1) Both periodic "background" and "source" or "defect" elements are now separately defined in translatable unit cells so that, in the algorithm, mutual electromagnetic interactions can be computed.

2) The near-interaction block matrix must allow for the possibility of "background-to-source" or "background-to-defect" cell interactions.

3) Matrices representing projections of both "background and source" or "background and defect" subdomain bases onto the interpolation polynomials must be defined and appropriately selected in forming the matrix-vector product.

It is important to note that, although here we consider a metamaterial layer with a dipole-antenna excitation, as per the extended GIFFT algorithm, "defect" elements could be considered as well.
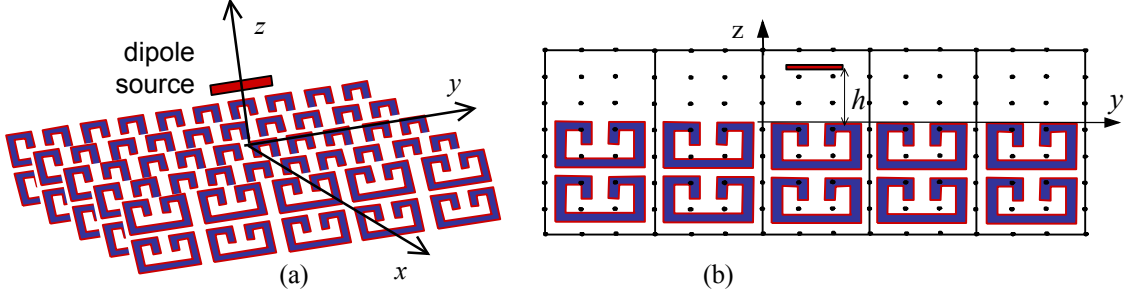

Fig. 1. (a) Typical geometry of the problems analyzed. A dipole antenna is placed over a periodic artificial material of finite size. A metamaterial structure is formed using two layers of capacitively-loaded split ring resonators (SRRs). The grid where the GF is evaluated in the y-z plane is represented in (b).

II. The GIFFT Method

A. Background: The GIFFT Method for Periodic Stuctures

In [1] the GIFFT method is applied to periodic structures (arrays, in particular) with polygonal boundaries. Only one element of the array is meshed and provided as input, while all other array elements are accounted for by taking advantage of the reusability properties of periodic structures comprising identical elements.

The GIFFT method begins by setting out a regular grid of Green's function interpolation points across the entire array. The points are typically chosen so there are four to six points per half-wavelength array cell (Fig. 1(b)). The points are used as equi-spaced interpolation nodes for Lagrange interpolating polynomials that approximate the Green's function as

$$\mathbf{G}(\mathbf{r} - \mathbf{r}') \cong \sum_{i,i'} L_i(\mathbf{r}) \, \mathbf{G}_{i-i'} \, L_{i'}(\mathbf{r}')$$

where $\mathbf{i}, \mathbf{i}'$ are double indices representing interpolation point locations overlaying the observation and source cells, respectively. The Green's function is sampled once for each unique value of the difference index $\mathbf{i} - \mathbf{i}'$ representing separation between source and observation interpolation points. It can be seen from the above that the Green's function approximation is of convolutional form, and a matrix-vector product involving it may utilize an FFT. After the Green's function is sampled, the basis functions are projected onto the interpolating polynomials. A correction is performed for neighboring elements by accounting for the interaction of a periodic cell with its neighbors via an accurate numerical integration. An iterative solver is then used that employs the FFT to perform the discrete convolution associated with the computation of matrix/vector products.

B. Modeling Sources

The GIFFT method requires that only distinct cell geometries be meshed and provided as input to the electromagnetic solver code. For very large structures this has the advantage of condensing the input data and reduces the chance of introducing mesh errors for complex structures. Thus, to model a dipole source over a finite metamaterial layer, we provide GIFFT the geometry for two distinct structures. The first consists of the mesh geometry for unit cells making up the "background" metamaterial layer. For the structure shown Fig. 1, the "background" unit cell can be taken as two split-ring resonators SRRs oriented along the z-direction. The second geometry description is that of the unit cell containing a *single dipole plus two SRR elements* (the "source" element). For the GIFFT technique, only the explicit meshing of these two distinct unit cells is required, with a replication of these "mother" cells automatically occurring in the computational part of the algorithm. (For the structure shown in Fig. 1, we have one "source" element and twenty-four "background" elements). It is significant to note that, for this implementation, the GF sampling grid has to be large enough to include both "background" and "source" elements (independently of their location) to form a large brick volume (only the y-z plane is shown in Fig. 1(b)) where the FFT algorithm is then applied.

III. Analysis Of A Dipole Over A High Impedance Surface

An finite size periodic material made of a two layer array of capacitively-loaded split-ring resonators (SRR) is studied here with a short strip dipole placed above the metamaterial at a height h above the top of the upper SRR, as shown in Fig. 1. The flat strip dipole is placed in the x-y plane (zero thickness along z) and is of width $W =$ 0.4 mm in the x-direction and length $L =$ 2.4 mm along the y-direction. It is fed by a delta-gap voltage generator at its center and meshed with three basis functions along its length. The SRRs geometry and arrangement have dimensions given by W = 4.06, L = 2.54, T = 0.457, U = 1.65, S = 1.245, G1 = 1.02, G2 = 0.508 (all in mm), where we have used the same notation as in [5]. As mentioned previously, throughout this study the basic unit cell (considered the "background" unit cell in the finite analysis) consists of two SRRs with the capacitive gaps facing the z-direction. A similar SRR-based metamaterial block, of both infinite and finite extent, has been studied by *Erentok et al.* in [5] and shown to provide an artificial-magnetic-conductor performance, with agreement between experiment and simulations being demonstrated. While in [5] these SRR elements were embedded in a duroid substrate of $\varepsilon_r = 2.2$, for a preliminary application of the GIFFT method an air substrate is here considered since it permits use of the FFT in all three dimensions. The periodicity along the x- and y-directions of the metamaterial layer are taken to be $a =$ 1.57 mm and $b =$ W + G1= 5.08 mm, respectively.

An analysis of memory requirements for the standard MoM method for the problem considering a metamaterial layer comprising (9×7 periodic elements), with 52 dof per element, shows that a Toeplitz storage format for the MoM impedance matrix requires 0.77×10^6 entries. In principle GIFFT requires the storage of only 84 GF samples per cell when we choose a $3 \times 7 \times 7$ points-per-cell interpolation scheme, so that a total of only 2.4×10^4 GF samples are stored. In practice, the memory requirement is slightly higher because of the zero padding to the nearest power of 2 needed to apply the FTT. Figure 10(a) shows how the size of the SRR substrate affects the input impedance of the short dipole located at a height $h =$ 2.5mm. Two cases have been considered: a small metamaterial substrate made of (7×3 periodic elements) and a large one made of (33×11 periodic elements). Also, for the large substrate case we have considered, the dipole height $h =$ 2 mm.

The trends of the real parts of the input impedance are not changed by the size, though the exact values do vary. The radiation patterns are shown in Fig. 10(b) for the small (7×3 periodic elements) and large (33×11) substrate cases with the dipole located at h = 2.5 mm at a frequency f = 13.73 GHz.
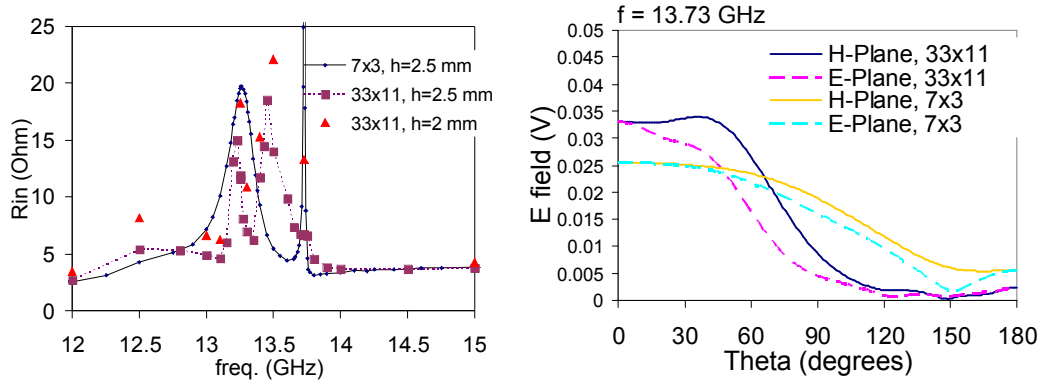

Fig. 2. Input Resistance for a short dipole located h=2.5 mm above a metamaterial substrate made of 7×3 and 33×11 cells (two SRRs in each cell). Also, the input impedance for a dipole with h= 2 mm is shown. (b) Radiation pattern of a dipole at a height h=2.5 mm, for the small (7×3 cells) and the large (33×11 cells) metamaterial substrate, at frequency f = 13.73 GHz.

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